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SELF-SIMILAR TRANSONIC FLOWS WITH AN OBLIQUE SHOCK WAVE

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Transonic self-similar flows with a shock wave in the form of a generalized parabola $\bar{x} / \bar{y}^{-n} = \text{const}$ ($\bar{x}$ and $\bar{y}$ are Cartesian coordinates, and n is the self-similarity flow exponent) were studied for the first time in [1, 2]. In this paper we consider the conditions under which the shock wave is rectilinear. Along with the case of a normal shock (for which examples were constructed in [3, 4]) there is the possibility of an oblique shock, wherein the property of shock wave and streamline being perpendicular at each point is not realized. We construct, as an example, a family of flows generalizing the results presented in [4].

Assuming that the flow velocity differs little from the sonic velocity, we can write the velocity vector components in the form [5]

$$\bar{v}_x = a_* (1 + \varepsilon v_x), \quad \bar{v}_{\bar{y}} = a_* \varepsilon^{3/2} v_y, \quad \bar{x} = x, \quad \bar{y} = \varepsilon^{-1/2} y \tag{1}$$

Here ε is a small quantity; a_* is the critical sound speed, and v_x and v_y are dimensionless components of a perturbation of the sonic flow which satisfy the equations

$$-v_x \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0, \qquad \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} = 0 \tag{2}$$

Self-similar solutions of the system (2) are of the form [6]

$$v_x = |y|^{2(n-1)} f(\xi), \quad v_y = |y|^{3(n-1)} g(\xi), \quad \xi = x |y|^{-n} \operatorname{sgn} y \tag{3}$$

When Eqs. (3) are substituted into (1), we obtain two ordinary differential equations [7]

$$f \frac{df}{d\xi} + n\xi \frac{dg}{d\xi} = 3(n-1) g, \quad n\xi \frac{df}{d\xi} + \frac{dg}{d\xi} = 2(n-1) f \tag{4}$$

The following conditions must be satisfied at the shock wave [2]:

$$f_1 + f_2 = 2n^2\xi^2, \quad g_2 - g_1 = -n\xi (f_2 - f_1), \quad \xi_1 = \xi_2 = \xi \tag{5}$$

The subscripts 1 and 2 refer to quantities on opposite sides of the shock front. An analysis of the relations (5) shows that shocks of the following types are possible:

1) a curved shock ($\xi \neq 0$, $n \neq 1$),
2) a rectilinear shock:
   a) $\xi = 0$, $g_1 = g_2 = 0$ for a normal shock,
   b) $\xi = 0$, $g_1 = g_2 \neq 0$ or $\xi \neq 0$, $n = 1$ for an oblique shock.

We can assume that the shock wave surrounding a local supersonic zone, which appears on a profile at large subsonic speeds, is an oblique shock. Using the flow scheme assumed in [4], we consider a set of flows of this type. To this end we investigate a self-similar Cauchy problem for the system (4). We specify the initial data on the back side of the shock front, i.e. for $\xi_2 = 0$ we can set f_2 equal to some value, which is constant for all cases, for example, the value -0.5. The values of the self-similarity exponent n and g_2 must satisfy the condition for the absence of limiting lines in Regions I and IV (see Fig. 1, where the solid curves denote streamlines; the region in which the Mach number $M<1$ is shown shaded). The solutions in Regions II and III are constructed by taking into account successive matching of values along the limiting characteristics C_i $(i=1,\ 2,\ 3)$. The shaded region in Fig. 2 shows the allowable values of n and g_2. In this region Frankl's example [4] corresponds to the point $n={}^3/_2$, $g_2=0$. The segment $g_2=0$ $({}^3/_2\leqslant n<2)$ corresponds to flows with a normal shock wave. When $g_2>0$ the tangential component of the velocity at the shock wave is directed towards

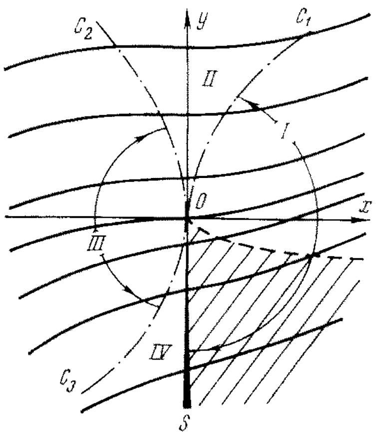

Fig. 1

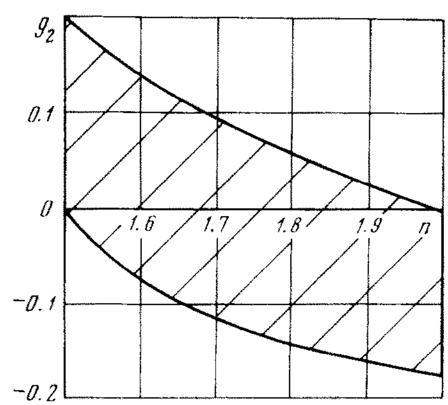

Fig. 2

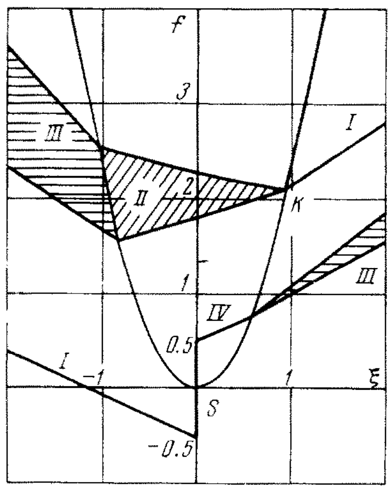

Fig. 3

the point of intersection of the shock with the sonic line. This indicates an attenuation of the shock wave at the point O (Fig. 1). Negative values of g_2, on the other hand, correspond to the case in which a shock arises at the sonic line; here in contrast to the situation in Fig. 1, the sonic line is situated in the first quadrant and is tangent to the shock wave.

On the limiting characteristics the flow is not analytic. We note that only when $n={}^3/_2$ does the assignment of the initial data f_2 and g_2 uniquely determine the flow in the neighborhood of the coordinate origin. It is uniquely determined for other values of n only in the Regions I and IV.

The calculated results for the set of flows considered are shown in Fig. 3 for $n=1.55$ and $g_2=0.1$. The totality of the integral curves, which describe all the possible flows in Region II, is bounded by curves (the image of the limiting characteristic C_1

forms the lower boundary (see Fig. 3) and that of C_2 forms the upper boundary) which are determined in a neighborhood of the point K by an asymptotic expansion of the form [8]

$$f = n^2 + d_1 (\xi - \xi_k)^2 + d_2 (\xi - \xi_k)^2 + \ldots$$

As g_2 approaches its maximum value (for a given value of n) the point K tends towards the parabola $f = n^2\xi^2$ at infinity; this means that the upper boundary of the region in Fig. 2 is not included in the set of allowable values.

The change in the angle of inclination of the streamline to the x-axis in the passage through the oblique shock wave is given by the expression

$$\theta_2 - \theta_1 = 2\varepsilon^{5/2} v_x v_y$$

It should be noted that in the class of self-similar solutions it is not possible to construct flows without limiting lines (limiting lines are present in the example given in [3]) and with a rectilinear shock wave, providing that a characteristic emanates from the end of the shock.

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Editor's Note. This reference is erroneously given in the Russian original. It does not appear in the PMM Journal indicated.